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Estimators of Repeatability for Perennial Crops

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Summary

New procedures for estimation of repeatability, which are robust to bienniality, are proposed and their efficiencies compared with the traditional methods of estimation based on ANOVA and principal components. A simulation study conducted under the standard linear model and with fixed and random biennial effects indicated the superiority of Moving Average I and II estimators over the other traditional methods based on ANOVA and principal components. Overall, the Moving average II is the most preferred method as it gives the least bias and mean squared error for entire range of repeatability, sample size, fixed and random biennial effect and for different intensities of 'on' and 'off' phase trees.

Key words : Robust to bienniality, Analysis of variance, Principal components, Moving average estimators, Monte carlo biases.

Introduction

Most perennial species, according to Pearce [3] are to some extent biennial in cropping and growth. The trees which have acquired this biennial rhythm carry a heavy crop in one year (called the 'on' tree) and little or no crop in the next year (the 'off' tree). The alternation of too much and too little crop in the 'on' and 'off' years respectively, may persist with great regularity, though it may be upset by some major climatic factors. The problem becomes more complex because of presence of both kinds of trees in the field at a given point of time. Due to presence of this tendency in perennial crops the usual statistical methods for estimation of genetic parameters become biased and less efficient. One of the important genetic parameters which does not need raising a progeny for its estimation, is the repeatability. There is not much work done for the efficient estimation of repeatability in perennial crops except for a few passing references of Abeywardena [1] and Rutledge [4]. It has been shown by these workers that in the presence of biennial rhythm in the data the different procedures for estimation of repeatability become highly biased due to confounding of variance due to bienniality with the error variance. There is no procedure available to estimate the repeatability in perennial crops which is robust to bienniality. In the present study new procedures which are robust to bienniality are proposed and their performance is compared with the traditional procedures based on analysis of variance, principal components and structural analysis.

2. *Methodology*

Two new estimators based on moving averages of two consecutive years are proposed to estimate repeatability in the presence of biennial rhythm.

3. *Moving Average Estimator-I*

Assuming standard linear model for describing the yield of an individual tree along with a fixed biennial effect 'b', the model is

$$Y_{ij} = \mu + g_i + t_j \pm b + e_{ij} \quad i = 1, 2, \ldots n \qquad (1)$$

$$j = 1, 2, \ldots k$$

where Y_{ij} = is the yield of ith tree for jth year

μ = is the overall mean

g_i = is the random effect of ith individual tree

t_j = is a fixed effect of jth year

e_{ij} = random error.

$g_i \sim N(0, \sigma_g^2)$, $e_{ij} \sim N(0, \sigma_e^2)$

We have suppressed the fixed year effect in the model (1) for the present study and the reduced model for the yield Y_{ij} is then

$$Y_{ij} = \mu + g_i \pm b + e_{ij} \qquad (2)$$

Taking the moving average of the two consecutive years, the fixed biennial effect will be eliminated but at the same time the errors of the corrected yield will become correlated with the following simple structure.

$$Y_{ij}(c) = (Y_{ij} + Y_{ij+1})/2 = \mu + g_i + \frac{(e_{ij} + e_{ij+1})}{2} \qquad (3)$$

and the $E(MSE) = \frac{1}{2}\,\sigma_e^2(1 - \bar{\rho})$ (4)

Where $\bar{\rho}$ is the correlation of the errors over the records in the corrected data and the covariance structure of the Error is

$$E(e_{ij}, e_{il}) = \begin{bmatrix} 1 & 0.5 & 0 & . & . & . & . & . & . & 0 & 0 \\ & 1 & 0.5 & . & . & . & . & . & . & 0 & 0 \\ & & 1 & . & . & . & . & . & . & 0 & 0 \\ & & & & & & & . & . & . & 0 & 0 \\ & & & & & & & & . & . & 0 & 0 \\ & & & & & & & & & . & 1 & 0.5 \\ & & & & & & & & & & & 1 \end{bmatrix} \sigma^2_{\bar{e}} \qquad (5)$$

$i = 1, 2, \ldots n \ \& \ j, l = 1, 2, \ldots k-1.$ and $\sigma^2_{\bar{e}} = \dfrac{k-2}{2(k-1)} \sigma^2_e$.

The total number of records of each tree after taking the moving average of two records as per (3) will be reduced by one. Now the estimate of error variance of the standard linear model as obtained from the mean square error of ANOVA carried as per model (3) will be equal to

$$\hat{\sigma}^2_e = \frac{2(k-1)}{(k-2)} \text{MSE} \qquad (6)$$

The estimate of variance of g_i component as obtained from the ANOVA carried on model (3) is :

$$\hat{\sigma}^2_g = (\text{MSG}-\text{MSE})/(k-1) \qquad (7)$$

The newly proposed estimator of repeatability based on (6) and (7) is the Moving Average-I estimate (MA-I) and is as follows :

$$\hat{\rho}_{MA-I} = \frac{(\text{MSG}-\text{MSE})}{\text{MSE} + \left\{ \dfrac{2(k-1)^2}{(k-2)} - 1 \right\} \text{MSE}} \qquad (8)$$

where MSG and MSE are the mean squares for individuals and error respectively. The formula (8) for large values of k can be approximated as :

$$\hat{\rho}_{MA-I} = \frac{\text{MSG}-\text{MSE}}{\text{MSG} + 2\,(k-1)\text{MSE}} \qquad (9)$$

4. *Moving Average Estimator-II*

The expected values of error mean square and the mean square between individuals based on model (3) are worked out by induction and are as follow :

$$E(\text{MSE}) = \frac{k-2}{2(k-1)} \sigma^2_e \qquad (10)$$

$$E(MSG) = (k-1)\,\sigma_g^2 + \frac{2k-3}{2(k-1)}\,\sigma_e^2 \tag{11}$$

where MSE and MSG are mean squares of error and between individuals based on model (3). The σ_e^2 is the error variance of the standard linear model in the absence of bienniality. The estimates of σ_g^2 and σ_e^2 can be obtained from (10) and (11) and these are :

$$\hat{\sigma}_e^2 = \frac{2(k-1)}{k-2}\,MSE \tag{12}$$

$$\hat{\sigma}_g^2 = \frac{1}{k-1}\left[MSG - \frac{2k-3}{k-2}\,MSE \right] \tag{13}$$

The estimator of repeatability obtained from these estimates in (12) and (13) is the Moving Average-II (MA-II) estimator and is as follow :

$$\hat{P}_{MA-II} = MSG - \left(\frac{2k-3}{k-2}\right) MSE/(MSG + \frac{1}{k-2}\,[(k-2)^2 + (k-1)^2]\,MSE) \tag{14}$$

The performance of these two estimators is compared with traditional estimator from Analysis of variance (ANOVA), Principal components of covariances and correlations i.e. (PC–COV and PC–COR) and the structural estimator from correlation matrix (STR-COR). The details of these existing estimators can be obtained from Mansour et al [2].

The data for comparing the performance of different estimators was simulated by fixing the population parameter of repeatability and variance as unity. Based on these values the corresponding σ_g^2 and σ_e^2 are obtained. Using these values of σ_g^2 and σ_e^2 the observations y_{ij} are obtained as per model (2) after adding/substracting the fixed/random biennial effect (b) based on the percentage of 'on' and 'off' phase trees to be taken at a particular year.

5. *Results and Discussion*

The Monte carlo biases and mean square errors of the six different estimators of repeatability with various levels of fixed bienniality as well as random bienniality under standard linear model for low, moderate and high values of population parameter are summarized in Table 1–6. These results are based on the 100 simulation runs. The traditional ANOVA estimator (Table 1) shows the least bias and mean square error in the absence of biennial effect. The bias and mean square error of the estimator increases many fold with the introduction of bienniality in the model. The estimators based on PC-COV and PC-COR are also biased and give higher bias and mean square error for low repeatability and almost comparable bias and mean square error for moderate and higher values of repeatability in the absence of bienniality. In the absence of bienniality these three estimators are highly biased and had given very high mean square error too. The estimator based on STR-COR has behaved almost similar to the ANOVA estimator uniformly for all the cases.

The newly proposed estimators based on moving averages i.e. MA-I and II (Table 5–6) are found to be robust to bienniality and gave almost comparable bias and mean square errors with various levels of fixed and random bienniality. The variation in the percentage of 'off' trees in the field has not affected the moving average estimators, whereas in the rest of the estimators there was a decline in the biases and mean square errors with increase in the percentage of 'off' trees. An increase in the magnitude of the fixed biennial effect has also resulted in higher biases and mean square errors in all the estimators except for the moving average estimators. There was a considerable decline in the biases and mean square errors of all the estimators with increase in the sample size i.e. both in the number of individuals and number of records. The increase in the number of records (k) was found more effective in reducing the baises and mean square errors as compared to the number of individuals (n).

Considering all the situations, that is, the various sample sizes, various levels of bienniality both fixed as well as random and different percentage of 'off' phase trees at a given point of time, the biases and mean square errors of moving average estimators are found to be least as compared to the other estimators. In the absence of bienniality the bias and mean square error of MA-II estimator is almost comparable to ANOVA estimator in case of large samples. The MA-I estimator gave comparable results to MA-II estimator except for low values of population repeatability when it gave slightly higher bias and mean square error. Based on these results, it can be concluded that the MA-II estimator is the best among these estimators and can be used with advantage both with and without bienniality in the data when the sample size is sufficiently large.

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Table 1. Monte Carlo bias (in 10^{-2}), and mean square error (in 10^{-4}) of ANOVA estimators of repeatability from standard linear model with & without bienniality for different values of ρ, proportion of 'off' trees, no. of trees (n) & no. of years (k)

ρ	off trees		0.05				0.50				0.90			
Bienniality (n, k)	%		(25, 3)	(25, 10)	100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)
WOB		B	.38	–.64	–.43	.01	–.83	–.12	–.14	–.42	–.80	.48	–.12	–.22
		M	146	19	27	4	110	67	37	15	15	10	2	3
WFB –± 3	50	B	–35.35	–14.48	–34.26	–14.46	–74.28	–54.90	–74.44	–55.06	–110.77	–90.12	–111.27	–90.96
		M	1263	209	1178	209	5545	3024	5546	3033	12304	8131	12388	8276
	90	B	–24.62	–11.15	–29.15	–12.17	–60.09	–43.88	–62.90	–44.76	–89.18	–71.30	–93.63	–75.08
		M	813	138	861	149	3906	2050	3990	2027	8570	5578	8830	5679
WRB	50	B	–30.64	–12.82	–30.81	–12.81	–66.20	–48.51	–65.30	–48.33	–98.95	–79.85	–97.51	–80.17
		M	973	165	958	164	4424	2365	4278	2338	9864	6405	9526	6434
	90	B	–19.91	–8.29	–20.74	–9.43	–48.84	–34.04	–47.72	–35.44	–69.11	–51.43	–69.75	–58.09
		M	493	85	454	91	2623	1272	2332	1289	5272	3082	4961	3441

Note : WOB-Without Bienniality, WFB- With Fixed Bienniality & WRB- With Random Bienniality

Table 2. Monte Carlo bias (in 10^{-2}), and mean square error (in 10^{-4}) of PC-COV estimators of repeatability from standard linear model with & without bienniality for different values of ρ, proportion of 'off' trees, no. of trees (n) & no. years (k)

ρ / Bienniality (n, k)	off trees %		0.05				0.50				0.90			
			(25, 3)	(25, 10)	100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)
WOB		B	17.05	10.26	5.62	3.01	1.77	2.22	.59	.17	–.36	–.04	–.02	–.12
		M	349	114	46	12	107	64	37	14	14	8	2	2
WFB –± 3	50	B	85.42	85.49	85.08	85.01	38.74	39.87	38.56	39.61	–2.20	–1.30	–2.69	–.89
		M	7302	7310	7240	7227	1507	1593	1488	1570	16	10	9	2
	90	B	64.89	65.91	71.04	70.25	21.23	23.31	21.76	22.20	–17.31	–13.69	–20.25	–17.88
		M	4562	4697	5068	4999	586	669	514	556	440	300	467	365
WRB	50	B	75.20	74.58	74.37	74.19	27.32	29.30	25.97	28.18	–14.44	–11.66	–15.85	–12.49
		M	5677	5567	5535	5506	769	869	682	797	247	161	263	163
	90	B	51.25	50.43	50.85	52.88	4.89	8.23	2.40	6.43	–31.58	–26.76	–37.91	–34.36
		M	2906	2843	2662	2843	163	215	57	106	1175	830	1485	1217

Note : WOB-Without Bienniality, WFB- With Fixed Bienniality & WRB- With Random Bienniality

Table 3. Monte Carlo bias (in 10^{-2}), and mean square error (in 10^{-4}) of PC-COR estimators of repeatability from standard linear model with & without bienniality for different values of ρ, proportion of 'off' trees, no. of trees (n) & no. years (k)

ρ / Bienniality (n, k)	off trees %		0.05				0.50				0.90			
			(25, 3)	(25, 10)	(100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)
WOB		B	11.82	8.16	3.51	2.35	.06	1.01	.13	−.17	−.44	−.12	−.04	−.14
		M	191	73	24	8	106	65	36	14	14	9	2	2
WFB −± 3	50	B	85.33	85.42	85.05	85.99	38.55	39.74	38.47	39.58	−2.42	−1.59	−2.69	−.94
		M	7286	7299	7235	7224	1493	1583	1481	1567	16	11	9	2
	90	B	64.09	65.39	70.94	70.16	20.43	22.65	21.64	22.03	−18.04	−14.91	−20.07	−18.17
		M	4483	4641	5053	4987	566	645	509	550	446	354	446	377
WRB	50	B	74.78	74.27	74.26	74.12	26.93	28.78	25.78	28.09	−14.92	−12.58	−16.08	−12.68
		M	5615	5521	5518	5496	746	839	671	792	252	186	268	168
	90	B	49.23	49.22	50.46	52.61	4.14	6.14	2.28	5.93	−32.56	−29.34	−37.44	−35.37
		M	2763	2738	2623	2816	143	206	051	103	1163	1008	1418	1290

Note : WOB-Without Bienniality, WFB- With Fixed Bienniality & WRB- With Random Bienniality

Table 4. Monte Carlo bias (in 10^{-2}), and mean square error (in 10^{-4}) of STR.COR estimators of repeatability from standard linear model with & without bienniality for different values of ρ, proportion of 'off' trees, no. of trees (n) & no. years (k).

ρ	off trees		0.05				0.50				0.90			
Bienniality (n, k)	%		(25, 3)	(25, 10)	100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)
WOB		B	.21	–.63	–.42	.00	–.86	.24	.04	–.34	–.45	–.15	–.04	.15
		M	151	19	28	4	111	70	37	15	14	9	2	2
WFB = ± 3	50	B	–34.85	–14.54	–34.49	–14.47	–75.09	–55.21	–75.00	–55.12	–111.40	–90.46	–111.42	–91.05
		M	1217	211	1190	209	5646	3051	5627	3039	12420	8193	12355	8292
	90	B	–24.69	–11.23	–29.16	–12.18	–59.90	–44.15	–62.92	–44.83	–89.28	–71.68	–93.38	–75.16
		M	789	139	856	150	3848	2078	3985	2034	8546	5641	8777	5692
WRB	50	B	–30.91	–12.92	–30.59	–12.84	–66.68	–48.89	–65.69	–48.39	–98.99	–80.35	–97.82	–80.27
		M	965	168	938	165	4465	2402	4317	2344	9836	6484	9579	6450
	90	B	–20.17	–8.40	–20.96	–9.44	–48.42	–34.41	–47.45	–35.52	–68.93	–51.47	–69.57	–58.14
		M	487	87	459	91	2567	1301	2297	1295	5204	3096	4939	3447

Note : WOB-Without Bienniality, WFB- With Fixed Bienniality & WRB- With Random Bienniality

Table 5. Monte Carlo bias (in 10^{-2}), and mean square error (in 10^{-4}) of MA-I estimators of repeatability from standard linear model with & without bienniality for different values of ρ, proportion of 'off' trees, no. of trees (n) & no. years (k)

ρ	off		0.05				0.50				0.90			
Bienniality (n, k)	trees %		(25, 3)	(25, 10)	100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)
WOB		B	19.43	4.26	–18.16	4.50	4.83	.55	6.03	1.47	–.39	–.25	–.12	–.40
		M	485	38	356	24	140	52	60	20	18	6	3	3
WFB = ± 3	50	B	18.28	4.40	17.84	4.75	5.06	–.15	4.22	.84	–.10	–.91	.23	–.18
		M	441	35	343	28	176	61	43	15	12	11	2	2
	90	B	20.17	4.15	18.22	4.97	5.33	–.98	5.32	1.13	–.23	–.61	.39	.21
		M	528	31	355	28	171	85	55	16	15	10	3	2
WRB	50	B	18.50	4.30	18.57	4.46	5.35	.33	6.22	1.30	–.49	–.45	.02	–.15
		M	439	34	375	24	122	78	72	16	20	10	3	2
	90	B	20.07	5.06	19.08	4.50	2.72	1.57	6.69	1.08	.11	–.62	.37	–.17
		M	515	44	389	25	134	55	71	23	13	11	3	2

Note : WOB-Without Bienniality, WFB- With Fixed Bienniality & WRB- With Random Bienniality

Table 6. Monte Carlo bias (in 10^{-2}), and mean square error (in 10^{-4}) of MA-II estimators of repeatability from standard linear model with & without bienniality for different values of ρ, proportion of 'off' trees, no of trees (n) & no. years (k)

ρ			0.05				0.50				0.90			
Bienniality (n, k)	off trees %		(25, 3)	(25, 10)	100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)	(25, 3)	(25, 10)	(100, 3)	(100, 10)
WOB		B	1.34	–.57	–.24	–.29	–1.33	–.88	.51	.12	–.72	–.31	–.39	–.46
		M	251	25	62	5	190	59	38	20	21	7	4	3
WFB – ± 3	50	B	–.43	–.40	–.71	–.01	–1.16	–1.62	–1.78	–.55	–.39	–.99	–.02	–.24
		M	250	19	57	6	249	71	45	17	13	11	3	2
	90	B	2.39	–.69	–.14	.23	–.80	–2.51	–.39	–.29	–.54	–.68	.15	.16
		M	278	18	55	4	241	101	44	17	17	11	3	2
WRB	50	B	–.04	–.52	.37	–.34	–.59	–1.13	.72	–.06	–.83	–.51	–.25	–.20
		M	220	20	71	6	151	89	53	16	24	10	3	2
	90	B	2.30	.32	1.18	–.30	–4.08	.20	1.34	–.29	–.18	–.69	.12	–.23
		M	257	22	60	6	236	59	44	24	15	11	3	2

Note : WOB-Without Bienniality, WFB- With Fixed Bienniality & WRB- With Random Bienniality